13 GRADE MATHEMATICS CURRICULUM

CAPE PURE MATHEMATICS

UNIT 2

- **UNIT 2: *COMPLEX NUMBERS*, ANALYSIS AND MATRICES**

  MODULE 1: *COMPLEX NUMBERS AND CALCULUS II*

(A) **Complex Numbers**

Students should be able to:

1. recognise the need to use complex numbers to find the roots of the general quadratic equation $ax^2 + bx + c = 0$, when $b^2 - 4ac < 0$;

2. *use the concept that complex roots of equations with constant coefficients occur in conjugate pairs;*

3. write the roots of the equation in that case and relate the sums and products to a, b and c;

4. calculate the square root of a complex number;

5. express complex numbers in the form $a + bi$ where a, b are real numbers, and identify the real and imaginary parts;

6. add, subtract, multiply and divide complex numbers in the form $a + bi$, where a and b are real numbers;

7. find the principal value of the argument θ of a non-zero complex number, where $-\pi < \theta \leq \pi$;

8. find the modulus and conjugate of a given complex number;

9. interpret modulus and argument of complex numbers on the Argand Diagram;

10. represent complex numbers, their sums, differences and products on an Argand diagram;

11. find the set of all points z (locus of z) on the Argand Diagram such that z satisfies given properties;

12. apply De Moivre's theorem for integral values of n;

13. *use* $e^{ix} = \cos x + i \sin x$, *for real* x.

(B) Differentiation II

Students should be able to:

1. find the derivative of $e^{f(x)}$, where f (x) is a differentiable function of x;

2. find the derivative of ln f (x) (to include functions of x – polynomials or trigonometric);

3. apply the chain rule to obtain gradients *and equations* of tangents and normals to curves given by their parametric equations;

4. use the concept of implicit differentiation, with the assumption that one of the variables is a function of the other;

5. differentiate any combinations of polynomials, trigonometric, exponential and logarithmic functions;

6. differentiate inverse trigonometric functions;

7. obtain second derivatives, $f''(x)$, of the functions in 3, 4, 5 above.

8. *find the first partial derivatives of* $u = f(x, y)$ *and* $w = f(x, y, z)$;

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(C) Integration II

Students should be able to:

1. express a rational function ***(proper and improper)*** in partial fractions in the cases where the denominators are:

   (a) distinct linear factors;

   (b) repeated linear factors;

(c) quadratic factors;

(d) repeated quadratic factors;

(e) combinations of (a) to (d) above (repeated factors will not exceed power 2);

2. express an improper rational function as a sum of a polynomial and partial fractions;

3. integrate rational functions in Specific Objectives 1 and 2 above;

4. integrate trigonometric functions using appropriate trigonometric identities;

5. integrate exponential functions and logarithmic functions;

6. find integrals of the form $\int \frac{f'(x)}{f(x)} dx$;

7. use substitutions to integrate functions (the substitution will be given in all but the most simple cases);

8. use integration by parts for combinations of functions;

9. integrate inverse trigonometric functions;

10. derive and use reduction formulae to obtain integrals;

11. use the trapezium rule as an approximation method for evaluating the area under the graph of the function.

UNIT 2
MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS

(A) Sequences

Students should be able to:

1. define the concept of a sequence $\{a_n\}$ of terms a_n as a function from the positive integers to the real numbers;

2. write a specific term from the formula for the n^{th} term, or from a recurrence relation;

3. describe the behaviour of convergent and divergent sequences, through simple examples;

4. apply mathematical induction to establish properties of sequences.

(B) Series

Students should be able to:

1. use the summation (Σ) notation;

2. define a series, as the sum of the terms of a sequence;

3. identify the n^{th} term of a series, in the summation notation;

4. define the m^{th} partial sum S_m as the sum of the first m terms of the sequence, that is, $S_m = \sum_{r=1}^{m} a_r$;

5. apply mathematical induction to establish properties of series;

6. find the sum to infinity of a convergent series;

7. apply the method of differences to appropriate series, and find their sums;

8. use the Maclaurin theorem for the expansion of series;

9. *use the Taylor theorem for the expansion of series.*

(C) The Binomial Theorem

Students should be able to:

1. explain the meaning and use simple properties of $n!$ and $\binom{n}{r}$, that is, ${}^nC_r$, where $n, r \in \mathbb{Z}$;

2. *recognise that ${}^nC_r$ that is, $\binom{n}{r}$, is the number of ways in which r objects may be chosen from n distinct objects;*

3. expand $(a + b)^n$ for $n \in \mathbb{Q}$;

4. apply the Binomial Theorem to real-world problems, for example, in mathematics of finance, science.

(D) Roots of Equations

Students should be able to:

1. test for the existence of a root of $f(x) = 0$ where f is continuous using the Intermediate Value Theorem;

2. *use interval bisection to find an approximation for a root in a given interval;*

3. *use linear interpolation to find an approximation for a root in a given interval;*

4. explain, in geometrical terms, the working of the Newton-Raphson method;

5. use the Newton-Raphson method to find successive approximations to the roots of $f(x) = 0$, where f is differentiable;

6. *use a given iteration to determine a root of an equation to a specified degree of accuracy.*

UNIT 2
MODULE 3: COUNTING, MATRICES AND *DIFFERENTIAL EQUATIONS*

(A) Counting

Students should be able to:

1. state the principles of counting;
2. find the number of ways of arranging n distinct objects;
3. find the number of ways of arranging n objects some of which are identical;
4. find the number of ways of choosing r distinct objects from a set of n distinct objects;
5. identify a sample space;
6. *identify* the numbers of possible outcomes in a given sample space;
7. *use Venn diagrams to illustrate the principles of counting;*
8. *use possibility space diagram to identify a sample space;*
9. *define* and calculate P(A), the probability of an event A occurring as the number of possible ways in which A can occur divided by the total number of possible ways in which all equally likely outcomes, including A, occur;
10. use the fact that $0 \leq P(A) \leq 1$;
11. demonstrate and use the property that the total probability for all possible outcomes in the sample space is 1;
12. use the property that $P(A') = 1 - P(A)$ is the probability that event A does not occur;
13. use the property $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for event A and B;
14. use the property $P(A \cap B) = 0$ or $P(A \cup B) = P(A) + P(B)$, where A and B are mutually exclusive events;
15. use the property $P(A \cap B) = P(A) \times P(B)$, where A and B are independent events;
16. *use the property* $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ *where* $P(B) \neq 0$.
17. *use a tree diagram to list all possible outcomes for conditional probability.*

(B) Matrices and Systems of Linear Equations

Students should be able to:

1. operate with conformable matrices, carry out simple operations and manipulate matrices using their properties;
2. evaluate the determinants of $n \times n$ matrices, $1 \leq n \leq 3$;

3. reduce a system of linear equations to echelon form;

4. row-reduce the augmented matrix of an $n \times n$ system of linear equations, $n = 2, 3$;

5. determine whether the system is consistent, and if so, how many solutions it has;

6. find all solutions of a consistent system;

7. invert a non-singular 3×3 matrix;

8. solve a 3×3 system of linear equations, having a non-singular coefficient matrix, by using its inverse.

(C) *Differential Equations and Modeling*

Students should be able to:

1. *solve first order linear differential equations* $y' - ky = f(x)$ *using an integrating factor, given that* k *is a real constant or a function of* x*, and* f *is a function;*

2. *solve first order linear differential equations given boundary conditions;*

3. *solve second order ordinary differential equations with constant coefficients of the form*

$$ay'' + by' + cy = 0 = f(x), \text{ where } a, b, c \in \mathbb{R} \text{ and } f(x) \text{ is:}$$

   (a) *a polynomial,*
   (b) *an exponential function,*
   (c) *a trigonometric function;*

   and the complementary function may consist of

   (a) *2 real and distinct roots;*
   (b) *2 equal roots;*
   (c) *2 complex roots.*

4. *solve second order ordinary differential equation given boundary conditions;*

5. *use substitution to reduce a second order ordinary differential equation to a suitable form.*