12 GRADE MATHEMATICS CURRICULUM

CAPE PURE MATHEMATICS

UNIT 1

- **UNIT 1: ALGEBRA, GEOMETRY AND CALCULUS**

MODULE 1: BASIC ALGEBRA AND FUNCTIONS

(A) Reasoning *and Logic*

Students should be able to:

1. *identify simple and compound propositions;*
2. *establish the truth value of compound statements using truth tables;*
3. *state the converse, contrapositive and inverse of a conditional (implication) statement;*
4. *determine whether two statements are logically equivalent.*

(B) The Real Number System – $\mathbb{R}$

Students should be able to:

1. *perform binary operations;*
2. *use the concepts of identity, closure, inverse, commutativity, associativity, distributivity addition, multiplication and other simple binary operations;*
3. *perform operations involving surds;*
4. construct simple proofs, specifically direct proofs, or proof by the use of counter examples;
5. establish simple proofs by using the principle of mathematical induction.

(C) *Algebraic Operations*

Students should be able to:

1. *apply the Remainder Theorem;*
2. use the Factor Theorem to find factors and to evaluate unknown coefficients;
3. extract all factors of $a^n - b^n$ for positive integers $n \leq 6$;
4. use the concept of identity of polynomial expressions.

(D) *Exponential and Logarithmic Functions*

Students should be able to:

1. *define an exponential function* $y = a^x$ *for* $a \in \mathbb{R}$;

2. *sketch the graph of* $y = a^x$;

3. *define a logarithmic function as the inverse of an exponential function;*

4. *define the exponential functions* $y = e^x$ *and its inverse* $y = \ln x$, *where* $\ln x \equiv \log_e x$;

5. *use the fact that* $y = \ln x \Leftrightarrow x = e^y$;

6. *simplify expressions by using laws of logarithms;*

7. *use logarithms to solve equations of the form* $a^x = b$;

8. solve problems involving changing of the base of a logarithm.

(E) Functions

Students should be able to:

1. *define mathematically the terms: function, domain, range, one-to-one function (injective function), onto function (surjective function), many-to-one, one-to-one and onto function (bijective function), composition and inverse of functions;*

2. *prove whether or not a given simple function is one-to-one or onto and if its inverse exists;*

3. *use the fact that a function may be defined as a set of ordered pairs;*

4. *use the fact that if* g *is the inverse function of* f, *then* $f[g(x)] = x$, *for all* x, *in the domain of* g;

5. illustrate by means of graphs, the relationship between the function $y = f(x)$ given in graphical form and $y = |f(x)|$ and the inverse of $f(x)$, that is, $y = f^{-1}(x)$.

(F) *The Modulus Function*

Students should be able to:

1. *define the modulus function;*

2. *use the properties:*

   (a) $|x|$ *is the positive square root of* x^2;

   (b) $|x| < |y|$ *if, and only if,* $x^2 < y^2$;

   (c) $|x| < |y| \Leftrightarrow iff\ -y < x < y$;

   (d) $|x+y| \leq |x| + |y|$.

3. *solve equations and inequalities involving the modulus function, using algebraic or graphical methods.*

(G) *Cubic Functions and* Equations

Students should be able to use the relationship between the *sum of the roots, the product of the roots, the sum of the product of the roots pair-wise* and the coefficients *of* $ax^3 + bx^2 + cx + d = 0$.

UNIT 1
MODULE 2: *TRIGONOMETRY,* GEOMETRY AND VECTORS

(A) Trigonometric Functions, Identities and Equations (all angles will be assumed to be in radians unless otherwise stated)

Students should be able to:

1. *use compound-angle formulae;*

2. *use the reciprocal functions of* $\sec x$, $\operatorname{cosec} x$ *and* $\cot x$;

3. *derive identities for the following:*

   (a) $\sin kA$, $\cos kA$, $\tan kA$, *for* $k \in \mathbb{Q}$;

   (b) $\tan^2 x$, $\cot^2 x$, $\sec^2 x$ and $\operatorname{cosec}^2 x$;

   (c) $\sin A \pm \sin B$, $\cos A \pm \cos B$.

4. *prove further identities using Specific Objective 3;*

5. express $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta \pm \alpha)$ and $r\sin(\theta \pm \alpha)$, where *r* is positive, $0 < \alpha < \frac{\pi}{2}$;

6. find the general solution of equations of the form:

   (a) $\sin k\theta = s$,

   (b) $\cos k\theta = c$,

   (c) $\tan k\theta = t$,

   (d) $a \cos \theta + b \sin \theta = c$,
   for $a, b, c, k, s, t, \in \mathbb{R}$;

7. *find the solutions of the equations in Specific Objectives 6 above for a given range;*

8. obtain maximum or minimum values of $f(a \cos \theta + b \sin \theta)$ for $0 \leq \theta \leq 2\pi$.

(B) Co-ordinate Geometry

Students should be able to:

1. find equations of tangents and normals to circles;

2. find the points of intersection of a curve with a straight line;

3. find the points of intersection of two curves;

4. obtain the Cartesian equation of a curve given its parametric representation;

5. obtain the parametric representation of a curve given its Cartesian equation;

6. *determine the loci of points satisfying given properties.*

(C) Vectors

Students should be able to:

1. *express a vector in the form* $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ *or* $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$
   where **i**, **j** *and* **k** *are unit vectors in directions of* x-, y- *and* z-*axis respectively;*

2. define equality of two vectors;

3. add and subtract vectors;

4. multiply a vector by a scalar quantity;

5. *derive and use unit vectors, position vectors and displacement vectors;*

6. find the magnitude and direction of a vector;

7. *find the angle between two given vectors using scalar product;*

8. *find the equation of a line in vector form, parametric form, Cartesian form, given a point on the line and a vector parallel to the line;*

9. *determine whether two lines are parallel, intersecting, or skewed;*

10. *find the equation of the plane, in the form* $x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = d$, $\mathbf{r}.\mathbf{n} = d$, *given a point in the plane and the normal to the plane.*

UNIT 1
MODULE 3: CALCULUS I

(A) Limits

Students should be able to:

1. use graphs to determine the continuity and discontinuity of functions;

2. describe the behaviour of a function f (x) as x gets arbitrarily close to some given fixed number, using a descriptive approach;

3. use the limit notation $\lim_{x \to a} \mathrm{f}(x) = L$, $\mathrm{f}(x) \to \mathrm{L}$ as $x \to a$;

4. *use the simple limit theorems:*
   If $\lim_{x \to a} \mathrm{f}(x) = F$, $\lim_{x \to a} \mathrm{g}(x) = G$ and k is a constant,

   then $\lim_{x \to a} k\mathrm{f}(x) = kF$, $\lim_{x \to a} \mathrm{f}(x)\mathrm{g}(x) = FG$, $\lim_{x \to a} \{\mathrm{f}(x) + \mathrm{g}(x)\} = F + G$,

   and, provided $G \neq 0$, $\lim_{x \to a} \frac{\mathrm{f}(x)}{\mathrm{g}(x)} = \frac{F}{G}$;

5. *use limit theorems in simple problems;*

6. *use the fact that* $\lim_{x \to 0} \frac{\sin x}{x} = 1$, *demonstrated by a geometric approach;*

7. *identify the point(s) for which a function is (un)defined;*

8. identify the points for which a function is continuous;

9. *identify the point(s) where a function is discontinuous;*

10. *use the concept of left-handed or right-handed limit, and continuity.*

(B) Differentiation I

Students should be able to:

1. *define the derivative of a function at a point as a limit;*

2. *differentiate, from first principles, functions such as:*

   (a) $f(x) = k$ *where* $k \in \mathbb{R}$

   (b) $f(x) = x^n$, *where* $n \in \{-3, -2, -1, -½, ½, 1, 2, 3\}$,

   (c) $f(x) = \sin x$,

   (d) $f(x) = \cos x$.

3. use the *sum*, product and quotient rules for differentiation;

4. differentiate *sums*, products and quotients of:

   (a) polynomials,

   (b) trigonometric functions;

5. *apply the chain rule in the differentiation of*

   (a) *composite functions (substitution),*

   (b) *functions given by parametric equations;*

6. *solve problems involving rates of change;*

7. use the sign of the derivative to investigate where a function is increasing or decreasing;

8. *apply the concept of stationary (critical) points;*

9. calculate second derivatives;

10. interpret the significance of the sign of the second derivative;

11. use the sign of the second derivative to determine the nature of stationary points;

12. *sketch graphs of polynomials, rational functions and trigonometric functions using the features of the function and its first and second derivatives (including horizontal and vertical asymptotes);*

13. describe the behaviour of such graphs for large values of the independent variable;

14. obtain equations of tangents and normals to curves.

(C) Integration I

Students should be able to:

1. *recognise integration as the reverse process of differentiation;*

2. demonstrate an understanding of the indefinite integral and the use of the integration notation $\int f(x)\, dx$;

3. show that the indefinite integral represents a family of functions which differ by constants;

4. demonstrate use of the following integration theorems:

    (a) $\int cf(x)dx = c\int f(x)dx$, where c is a constant,

    (b) $\int \{f(x) \pm g(x)\}\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

5. find:

    (a) indefinite integrals using integration theorems,

    (b) integrals of polynomial functions,

    (c) integrals of simple trigonometric functions;

6. integrate *using substitution;*

7. use the results:

    (a) $\int_a^b f(x)dx = \int_a^b f(t)dt$,

    (b) $\int_0^a f(x)dx = \int_0^a f(x-a)dx$ for $a > 0$,

    (c) $\int_a^b f(x)\, dx = F(b) - F(a)$, where $F'(x) = f(x)$;

8. apply *integration to:*

    (a) finding areas under the curve,

    (b) finding areas between two curves,

    (c) *finding volumes of revolution by rotating regions about both the x- and y-axes,*

9. *given a rate of change with or without initial boundary conditions:*

    (a) *formulate a differential equation of the form $y' = f(x)$ or $y'' = f(x)$ where f is a polynomial or a trigonometric function,*

    (b) *solve the resulting differential equation in* (a) *above and interpret the solution where applicable.*